Less Is More: Using Mathematical Conceptual Principles and Procedural Themes with Elementary Education Majors

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Abstract: Architect Ludwig Mies van der Rohe's "less is more" architectural motif describes an aesthetic approach of emphasizing a building's frame by reducing the edifice. This paper argues for a similar philosophical vision in the teaching of mathematics. This article describes such a different philosophical design for elementary teachers' content course – one that structures knowledge around major procedural themes, conceptual principles and facilitates learners to connect what they already know to new understandings. Results of 2700 students over 6 years show promise for a "less is more" methodology. Significant improvements occurred in procedural and conceptual knowledge, specifically with students' computational skills and the pass-rates success.

Key words: Elementary mathematics education; Pre-service teachers, Primary school teachers; Content knowledge; Conceptual principles; Procedural themes; Mathematics curriculum

Introduction

Architect Ludwig Mies van der Rohe (1959) espoused the maxim "Less is More" to describe his aesthetic approach of emphasizing a building's frame by reducing the structure to a strong, transparent, elegant casing. A similar philosophical vision is needed in the teaching of mathematics, specifically in the teaching of content courses for pre-service elementary education teachers.

There is an obvious need to teach for significant comprehension of mathematics in university elementary-education mathematics content courses (Committee on Prospering in the Global Economy of the 21st Century, 2006; The Glenn Commission, 2000; National Mathematics Advisory Panel, 2006). University pre-service elementary education majors enter elementary mathematics content courses with a fundamental lack of mathematics content knowledge, both conceptual and procedural (Ball, Hill, & Bass, 2005; Ma, 1999). In fact, two combined local studies (Zollman, 2002, 2003) show almost two-thirds (65% of the 459 students) are not adept at basic fifth-grade arithmetic computational skills. These undergraduates are our future classroom teachers.

Often, the mathematics content course is a review of the mathematics topics of the elementary school, beginning with the kindergarten topic of counting and advancing

to algebra. Lee Shuman (1999) argues current undergraduate mathematics programs have no time or interest in teaching for profound understanding. Shuman says programs misconstrue the elementary mathematics content courses as remedial rather than deservingly rigorous. The customary program for preparing elementary teachers requires one or two content courses in elementary mathematics.

Typically, this content course (or courses) has more than 60 separate topics in its review of elementary mathematics (Billstein, Libeskind, & Lott, 1997; Musser, Burger, & Peterson, 2003). This does not mean that all 60 plus topics are covered in the course. However, similar studies of public school teachers found the majority of teaching time was based directly on covering the textbook (Schmidt, McKnight, & Raizen, 1997). Teaching each as a separate topic as presented in the textbook, depth is not feasible. Schmidt, McKnights, and Raizen (1997) call this type of curriculum "a mile wide and an inch deep" (p. 122). Profound understanding could not happen, as there is never enough time to delve deeply into the content of elementary mathematics (Ma, 1999).

Less is More

The primary goal of this project was to give undergraduates a deep understanding of elementary-education mathematics content. Our project redesigned the elementary education mathematics content course through common major mathematical themes to reduce the total number of topics. The content was reorganized to help elementary education majors understand major mathematical connections of the content, fewer topics allow more depth of learning (Zollman, 2007).

Our approach, allowing for deeper understanding, is to cover, and combine, these individual topics by structure. This is the philosophy of this project. Most individual topics in the content curriculum have common conceptual similarities. For example, "how and why" we add algebraic expressions, decimals, and fractions is conceptually, and procedurally, the same as when we add whole numbers – we only add "like denominations." Similarly, the concept of using "unit-of-rate" procedures to solve proportions problems also is a mathematically correct method to solve percent problems, and also is a method to solve similar geometric figures situations.

Not only does this redesign "reduce" the separate topics, but also it allows time for a deeper understanding of the elementary mathematics content. This content is taught at a rigorous university level using the approach of mathematical connections. This approach is not a review of the elementary mathematics curriculum. This approach requires a deep understanding of the mathematics topics, both conceptual and procedural, to see the connections across the curriculum (Zollman, 2007).

Content Design: Two Major Principles and Five Main Themes

This project is based upon the National Research Council's (NRC) "Learning with Understanding: Seven Principles of Human Learning" (2002). These seven principles are:

1. Principled Conceptual Knowledge,

2. Prior Knowledge,

3. Metacognition,

4. Differences Among Learners,

5. Motivation,

6. Situated Learning, and

7. Learning Communities.

Specifically, this project addresses the first two principles: that learning is facilitated when new knowledge is structured around major concepts (NRC Principle 1); and that learning is facilitated when learners connect what they already know to construct new understandings (NRC Principle 2). These two principles deal with the design of the curriculum that is the focus of this paper. NRC principles 3 through 7 deal with the methods of instruction.

For Principle Conceptual Knowledge (NRC Principle 1), we structured and combined all the traditional elementary education course topics into the following five major mathematical "concept themes" from Lynn Steen's *On the Shoulders of Giants: New Approaches to Numeracy* (1990). [Note: similar categorization can be found in *Measuring Student Knowledge and Skills: A New Framework for Assessment* (1999), the Organisation (sic) for Economic Co-Operation and Development (OECD/PISA) that organizes the mathematics content around six major mathematical ideas.]

1. Change, e. g., to represent the passing from one state or stage to another;

2. Dimension, e. g., to specify the size or measure;

3. Quantity, e. g., to count, calculate, or express number sense;

4. Shape, e. g., to explain the geometric form or character;

5. Uncertainty, e. g., to describe the likelihood that an event will or will not occur.

Inherent in our design of each traditional curriculum topic is the specific connection to the concepts of the discipline and to the prior knowledge of the previous topics (NCR Principle 2, Prior Knowledge). Learners use what they already know to construct new understandings, both conceptual and procedural knowledge (Zollman, 2007). The following is an example of using prior knowledge to connect topics through the mathematical theme "dimension".

Connecting Topics through Mathematical Structure–Multiplication Represented as Rectangular Area: An Example.

Using the concept theme of "dimension" as in Steen (1990), we begin with **rectangular areas** to represent **whole number multiplication**. All of the following examples are from the course packet of the study (Math 201, 2000). This structure can extend to **fraction multiplication**, **decimal multiplication**, and **algebraic binomial expansion**. Rectangular-area structure can lead to solutions for **probability** problems, both **unconditional** and **conditional** representations.

For **whole number multiplication**, the exercise 3×5 we would have a rectangle 5 units length by 3 units width as shown below. The area of the rectangle is 15 square units, using color tiles.
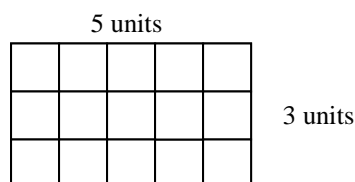
5 units

3 units

Extending this to utilize base-ten place value with base-ten blocks, 12×16 follows (see the diagram on next page). The partial products algorithm model shows the following:

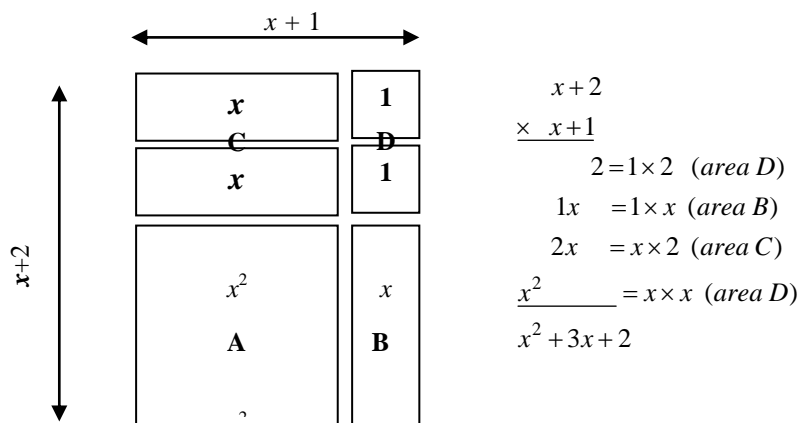|			
	12		$(10 + 2)$
	$\times\ 16$		$\times\ (10 + 6)$
(units, area D)	12	=	6×2
(vertical rods, area B)	60	=	6×10
(horizontal rods, area C)	20	=	10×2
(flats, area A)	$+\ 100$	=	10×10

192 square units for the total area
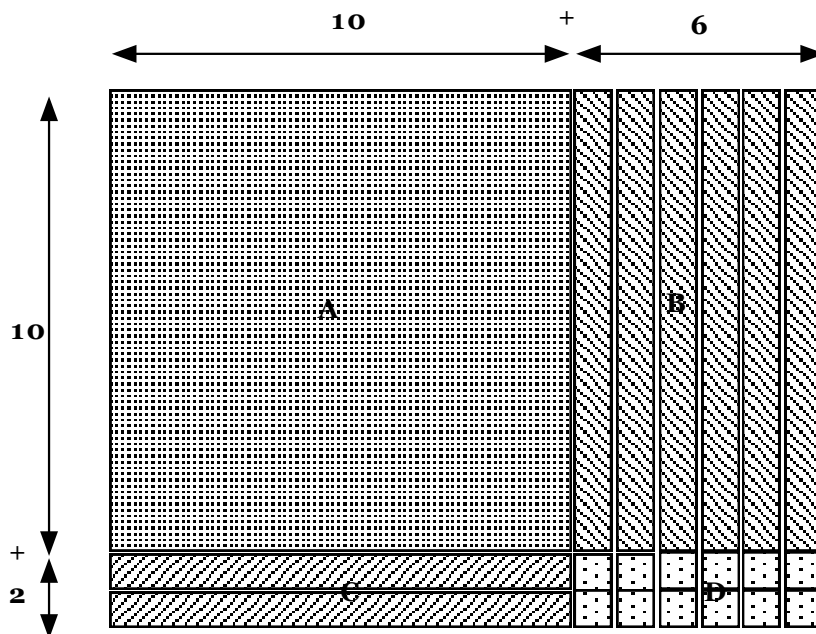
The area model for **fraction multiplication** is below with the similar partial products algorithm model.
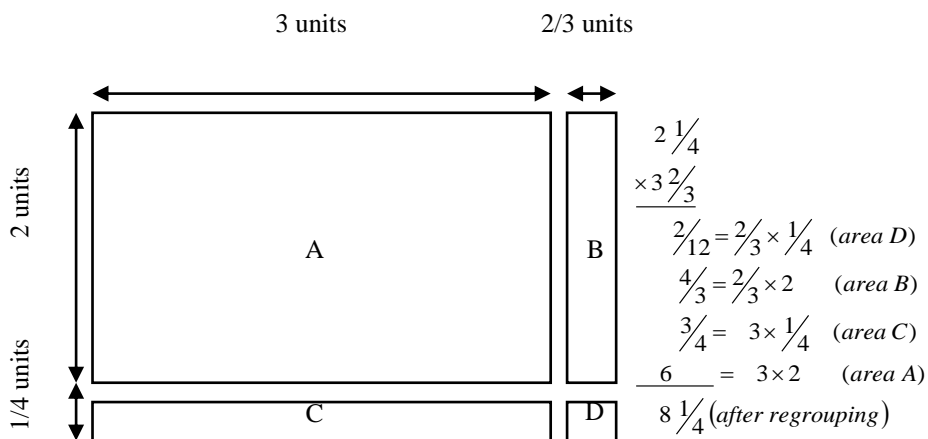
The structure for the area model for **decimal multiplication** is similarly shown, also with partial products algorithm model.

$$0.5$$
$$\times 1.5$$
$$0.25 = 0.5 \times 0.5 \;\; (\textit{area A})$$
$$\underline{0.50} = \;\; 1 \times 0.5 \;\; (\textit{area B})$$
$$0.75$$
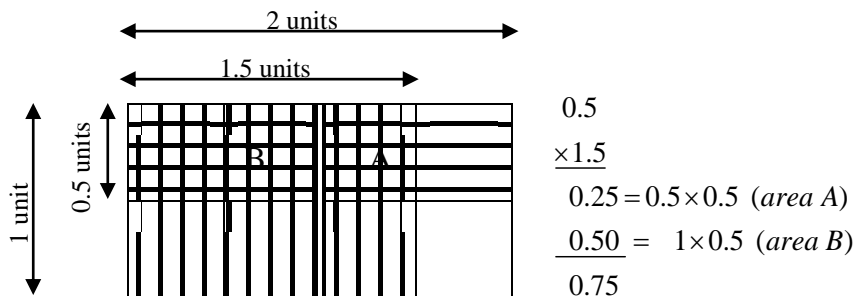
The area model for **binomial multiplication** also can have the same structure. And, it also can be shown with a partial products algorithm model.

$$x + 2$$
$$\times \;\; x + 1$$
$$2 = 1 \times 2 \;\; (\textit{area D})$$
$$1x = 1 \times x \;\; (\textit{area B})$$
$$2x = x \times 2 \;\; (\textit{area C})$$
$$\underline{x^2} = x \times x \;\; (\textit{area D})$$
$$x^2 + 3x + 2$$

Lastly, both conditional probability and unconditional probability also can revisit the multiplication as rectangular area structure. Examples for each are shown below.

Unconditional Probability Example: If DeShera's free throw percentage is 75%, what will be the probability that she makes exactly 1 shot of a two-shot foul shooting?

The shading in the first three columns represents DeShera making her second shot (p=3/4).

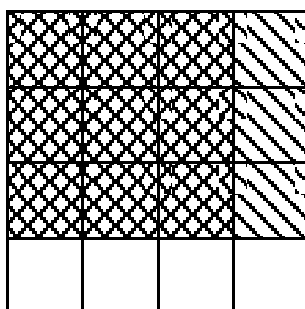*The shading in the first three rows represents DeShera making her first shot (p=3/4).*
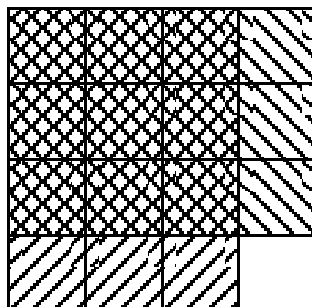
The boxes that are shaded above in only one direction represent DeShera making exactly 1 shot.

The correct answer is: 6/16 or (3/8 simplified).

Conditional Probability Example: If DeShera's free throw percentage is 75%, what will be the probability that she makes exactly 1 shot of a one-and-one foul shooting? Remember that to attempt the second shot DeShera must make her first shot.

The shading in the first three rows represents DeShera making her first shot.

If DeShera makes her first shot then the probability of her making her second shot is still 75%. So, we will shade in the first three columns, but we stop after three row.

*If DeShera misses her first shot, then she does not get a second shot. Thus, the probability of her making the second shot is **0**.*

The boxes that are shaded in only one direction represent DeShera making exactly 1 shot. The correct answer is $\frac{3}{16}$.

Methodology

Approximately 230 undergraduate students take the university's elementary education mathematics content course each fall and spring semester. A total of 2,721 students were used in this 6 year project. Normally, these students are taught in 8 sections, four 50-minute classes per week, for 15 weeks. Only full-time faculty teach this course (no graduate students nor adjunct faculty).

Over the three-year development of the curriculum, our project used information garnered through an evaluation of the traditional curriculum to redesign the organization of the content. A committee of mathematics department professors and instructors, experienced with the teaching of elementary mathematics education, performed regular monthly reviews on the outline of the content and the descriptions of correlated lessons during the first three summers of the curriculum development. This committee varied slightly from one summer to the next but normally consisted of three professors and three instructors. During the academic year, the project used three measures. The first was data collected on students' beliefs; the second was procedural (computational assessment) proficiency; third was a combination of the standardized departmental final exams, and course pass-rate success.

A second measure for the project was students' computational assessment results. Pre and post-tests of students' mathematical proficiency on computational assessment tests were conducted over a period of seven years.

As a third measure, control versus experimental groups were measured on: (a) pass-rate success (course grades of A, B, or C, vs. course grades of D, F, or W) and (b) standardized departmental exams. The control groups, comprised of the previous year's classes, used the traditional curriculum. The project's experimental groups used the redesigned curriculum of the correlated lessons (See Table 2).

Results

The computational assessment of paper-and-pencil computational operations was given to the students as a pre-test (first weeks of the semester) and as a post-test (during the departmental final exam). The results showed a significant improvement of the students' computational skills. Only 28% to 53% of the students were proficient on the pre-test; but 61% to 76% of the students who took the departmental final were proficient on the post-test (See Table 1). We defined proficient as obtaining at least 70% correct on basic operations of whole and

rational numbers. [Disclaimers: (1) During the semester the course populations decreased around 5% to 8% from student withdrawals. (2) The content of the computational skill test was not taught specifically. The topics were definitely a part of the course, developed to a deeper understanding. (3) To assure rigor, the departmental final exams for an 8 1/2 year period (1997 to 2005) were isomorphic in length, content, depth, and difficulty.] Despite these three disclaimers, most students did improve significantly in the course.

Table 1
Pre- & Post-Test Computational Assessments

Semester	Year	Number of Students	Pre-Test Proficiency	Post-Test Proficiency
Fall	2000	230	28.5%	76.1%
Spring	2001	223	25.5%	75.3%
Fall	2002	235	35.5%	63.7%
Spring	2003	236	40.6%	60.8%
Fall	2003	219	53.3%	64.3%
Spring	2004	239	51.4%	57.2%
Fall	2004	230	42.1%	57.1%
Spring	2005	220	41.9%	60.1%

Note: (1) Data is not available on some semesters. (2) Proficient: Percent of student population obtaining 70% or better on computational assessment.

 The project's main accomplishment is the success rates of students in the course over the past years. Here, the university's success rate is defined a course grade of A, B, or C versus a course grade of D, F, or W (withdrawal). The average success rate for the 2,721 students in the project was raised 12.7% in the past several years, compared to the previous years (see Table 2). For the latest group, fall semester 2005, the student success rate for the course was 79% for the 212 students. The experimental group mean of 76.7% is significant compared to the control group mean of 64%; to the university mean of 52% success rate for all general education mathematics courses at the university, and to the national mean success rate of 40%. Less content permitted more depth and understanding (Ball, Hill, & Bass, 2005).

Final Comments

Architect Ludwig Mies van der Rohe's "less is more" architectural motif (1959) describes an aesthetic approach of emphasizing a building's frame by reducing the edifice. This paper argues for a similar philosophical vision in the teaching of mathematics, specifically in the teaching of content courses, both concepts and procedures, for pre-service elementary education teachers.

Table 2
Student Success Rates on End-of-Semester Course Grades (A, B, or C vs. D, F, or W)

Control Population

Semester	Year	Number of Students	Success Rate
Fall	1997	175	58%
Spring	1998	226	60%
Fall	1998	219	62%
Spring	1999	239	74%
Fall	1999	238	64%
		MEAN	64.0%

Note: 64.0% of 1,097 students received a course grade of either an "A" or "B" or "C"

Experimental Population

Semester	Year	Number of Students	Success Rate
Spring	2000	234	79%
Fall	2000	230	67%
Spring	2001	223	77%
Fall	2001	231	71%
Spring	2002	212	82%
Fall	2002	235	77%
Spring	2003	236	72%
Fall	2003	219	79%
Spring	2004	239	80%
Fall	2004	230	83%
Spring	2005	220	75%
Fall	2005	212	79%
		MEAN	76.7%

Note: 76.7% of 2,721 students received a course grade of either an "A", or "B", or "C".

This project describes a different philosophical approach to the "Mathematics for Elementary Teachers" content course. First, the curriculum and instruction design is based on the National Research Council's (2002) Seven Principles of Human Learning: conceptual knowledge, prior procedural knowledge, metacognition, individual differences of learners, motivation, situated learning, and learning communities. For conceptual knowledge and prior procedural knowledge, our curriculum is designed into five conceptual themes, following Lynn Steen's *On the Shoulders of Giants: New Approaches to Numeracy* (1990): *change*, to represent the passing from one state or stage to another; *dimension*, to specify the size or measure; *quantity*, to count, calculate, or express number sense; *shape*, to explain the geometric form or character; and *uncertainty*, to describe the likelihood that an event will or will not occur.

This project also has several unique aspects. First, it organizes the content around the NRC's Principles of Human Learning (2002), not the traditional number system scope-and-sequence. Specifically it is arranged around the first two principles: that learning is facilitated when new knowledge is structured around major concepts (NRC Principle 1); and that learning is facilitated when learners connect what they already know to construct new understandings (NRC Principle 2). For example, "algebra sense" immediately follows "number sense" continuing the mathematical connections using the same method(s) of solution.

Second, the correlated lesson plans integrate proven instructional strategies of cooperative learning, connections to prior knowledge, manipulative use, learner reflections, multiple representations, situated learning, and outside study groups. These instructional strategies, hopefully, serve as a role model for their teaching of elementary students.

Third, the project gives emphasis to both conceptual and procedural knowledge by focusing mathematics as processes, not topics.

The long-term, yearly improvements of the students' success rates are not the sole result of the reorganization of the curriculum. Rather the improvements are the result of many factors seamlessly meshing. The reorganization and substantive reduction in the total number of topics allowed for: the teaching for deeper conceptual knowledge, connections to prior procedural knowledge, metacognition, individual differences of learners, motivation, situated learning, and learning communities. This unique philosophical approach initiates the change factors (Zollman, 2007).

As reform-based K-12 curriculum projects have found, the teacher and the teacher's implementation of the curriculum is the most important factor influencing success (National Research Council, 2004). It is the implementation by the instructors that is the critical factor of this project also. In this project, the instructors were imbedded in the development of the project, thus they felt ownership for its success. For similar success at other institutions, the instructors would need to be similarly committed to the project's success. A curricula and instructional reformation is necessary, but it is not sufficient.

The traditional elementary mathematics content approach of separately reviewing the 60 plus mathematics topics of the elementary school curriculum is not working. The majority of teaching time is based directly on covering the textbook (Schmidt, McKnight, & Raizen, 1997). Our future teachers need a college-level course that provides profound understanding of elementary mathematics. A course that plans knowledge structured around major concepts and facilitates learners to connect what they already know to new understandings can succeed in this endeavor.

References

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching. *American Educator, 29*(3), 14-22; 43-46.

Billstein, R., Libeskind, S., & Lott, J. W. (1997). *A problem solving approach to mathematics for elementary school teachers* (6th ed.). Reading, MA: Addison-Wesley.

Committee on Prospering in the Global Economy of the 21st Century: An Agenda for American Science and Technology, National Academy of Sciences, National Academy of Engineering, Institute of Medicine. (2006). *Rising above the gathering storm: Energizing and employing American for a brighter economic future.* Washington, DC: National Academies Press.

The Glenn Commission. (2000). *Before it's too late: A report to the nation from the National Commission on mathematics and science teaching for the 21st century.* Washington, DC: U.S. Department of Education.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates.

Math 201. (2000). *Foundations of elementary school mathematics (Math 201) course packet.* Department of Mathematical Sciences, Northern Illinois University, DeKalb, IL.

Musser, G. L., Burger, W. F., & Peterson, B. E. (2003). *Mathematics for elementary teachers: A contemporary approach* (6th ed.). New York: John Wiley & Sons, Inc.

National Mathematics Advisory Panel. (2006). *First meeting of the national mathematics advisory panel.* Washington, DC: U.S. Department of Education.

National Research Council. (2002). *Learning and understanding: Improving advanced study of mathematics and science in U.S. high schools.* Washington, DC: National Academies Press.

National Research Council. (2004). *On evaluating curricular effectiveness: Judging the quality of K-12 mathematics evaluations.* Committee for a review of the evaluation data on the effectiveness of NSF-supported and commercially generated mathematics curriculum materials. Mathematical Sciences Education Board, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

Organisation for Economic Co-Operation and Development. (1999). *Measuring student knowledge and skills: A new framework for assessment.* Paris: Author.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision: An investigation of U.S. science and mathematics education*, Dordrecht, The Netherlands: Kluwer.

Shuman, L. (1999). "Foreword" In L. Ma, *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Steen, L. A. (Ed.). (1990). *On the shoulders of giants: New approaches to numeracy*. Washington, DC: National Academy Press.

Mies van der Rohe, L. (1959, June 28). On restraint in design. *New York Herald Tribune.*

Zollman, A. (2002, March). *Making connections: Helping students reflect on the mathematics content.* Paper presented at the Research Council on Mathematics Learning, Memphis, TN.

Zollman, A. (2003, October). *Rocks in a jar: A structure approach for understanding elementary education mathematics content.* Paper presented at the 102nd Annual Conference of the School Science and Mathematics Association, Columbus, OH.

Zollman, A. (2007). Teaching for profound understanding: A study using mathematical connections with preservice elementary education majors. In D. F. Berlin & A. L. White (Eds.), *Global issues, challenges, and opportunities to advance science and mathematics education* (pp. 287-

304). Columbus, OH: International Consortium for Research in Science and Mathematics Education.

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